

Designing Math Teaching around Digital Manipulatives

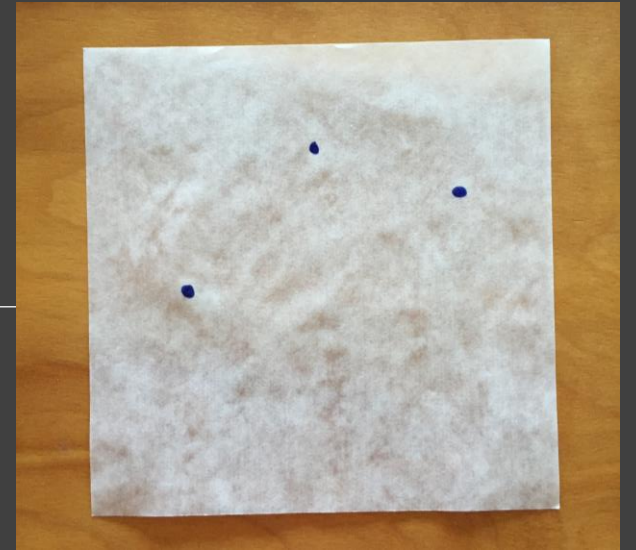
SCOTT FARRAR

OCTOBER 2015

What's a manipulative?

Lets try something...

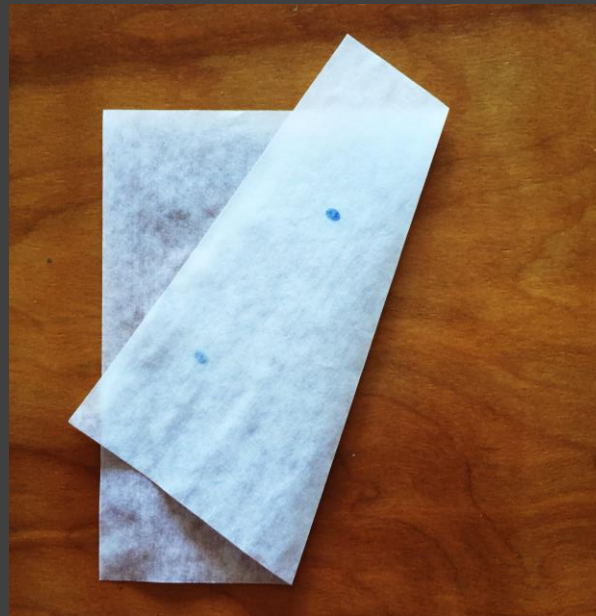
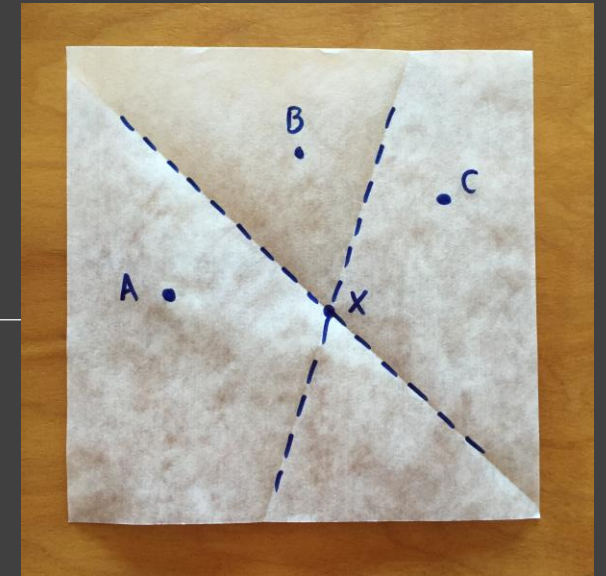
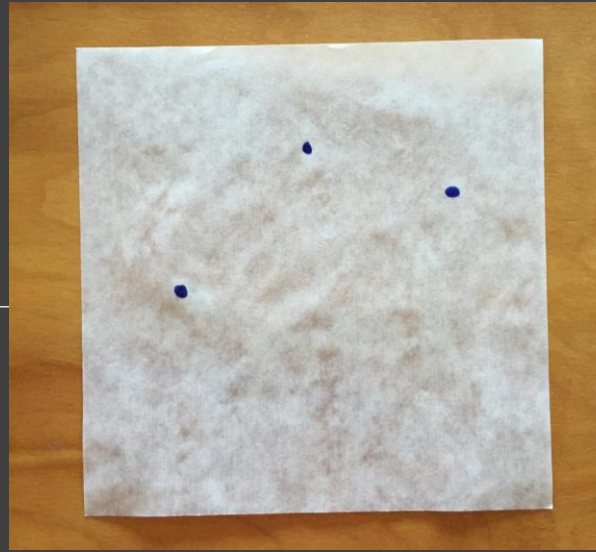
1. Place 3 random points on your "Patty Paper"
2. Fold two points "on top of each other" and crease the fold.



What's a manipulative?

3. Fold your 3rd point "on top of" either of the other points and crease it.

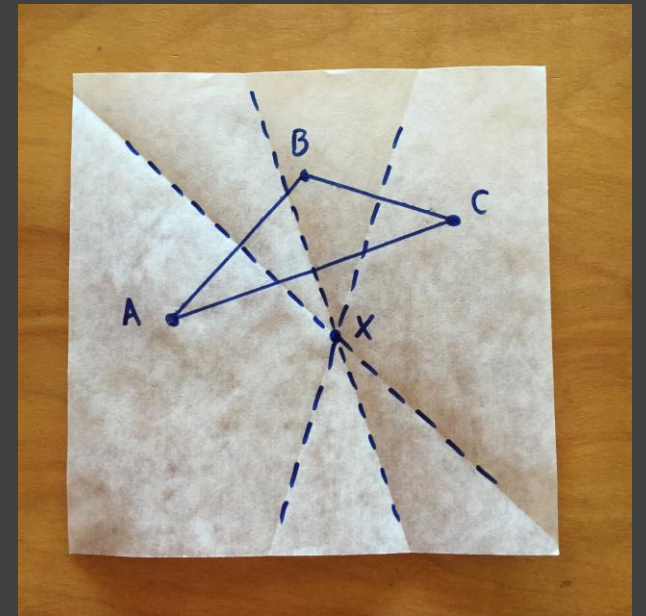
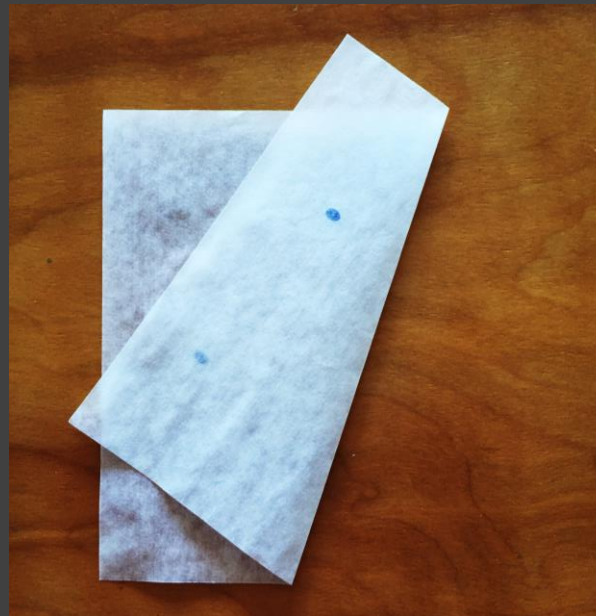
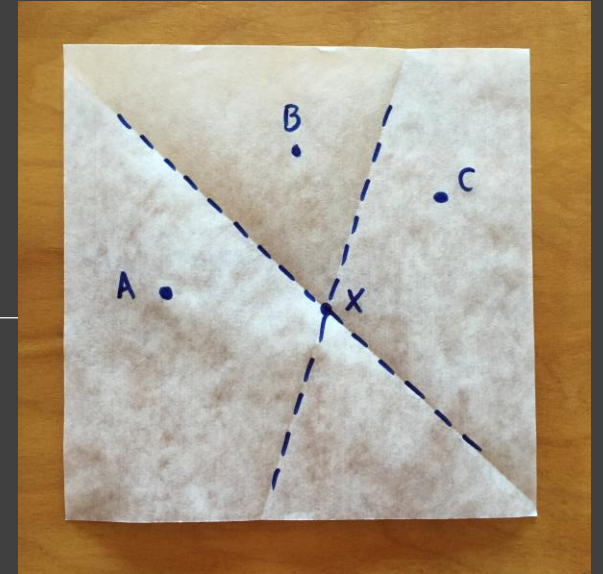
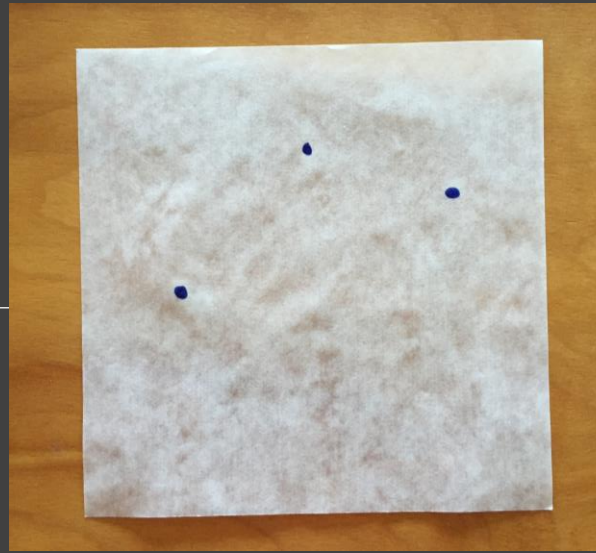
4. Where do your folds cross? What is special about it?



What's a manipulative?

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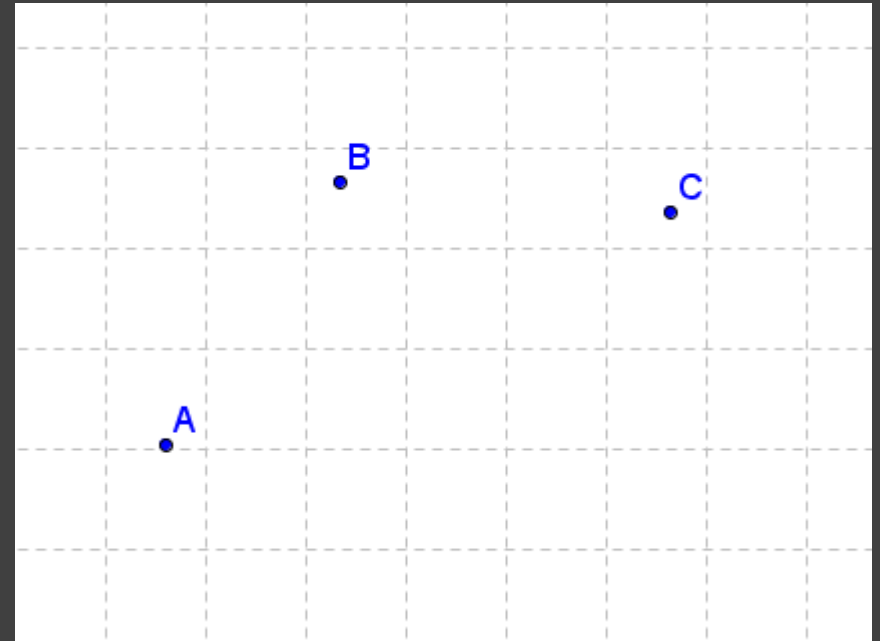


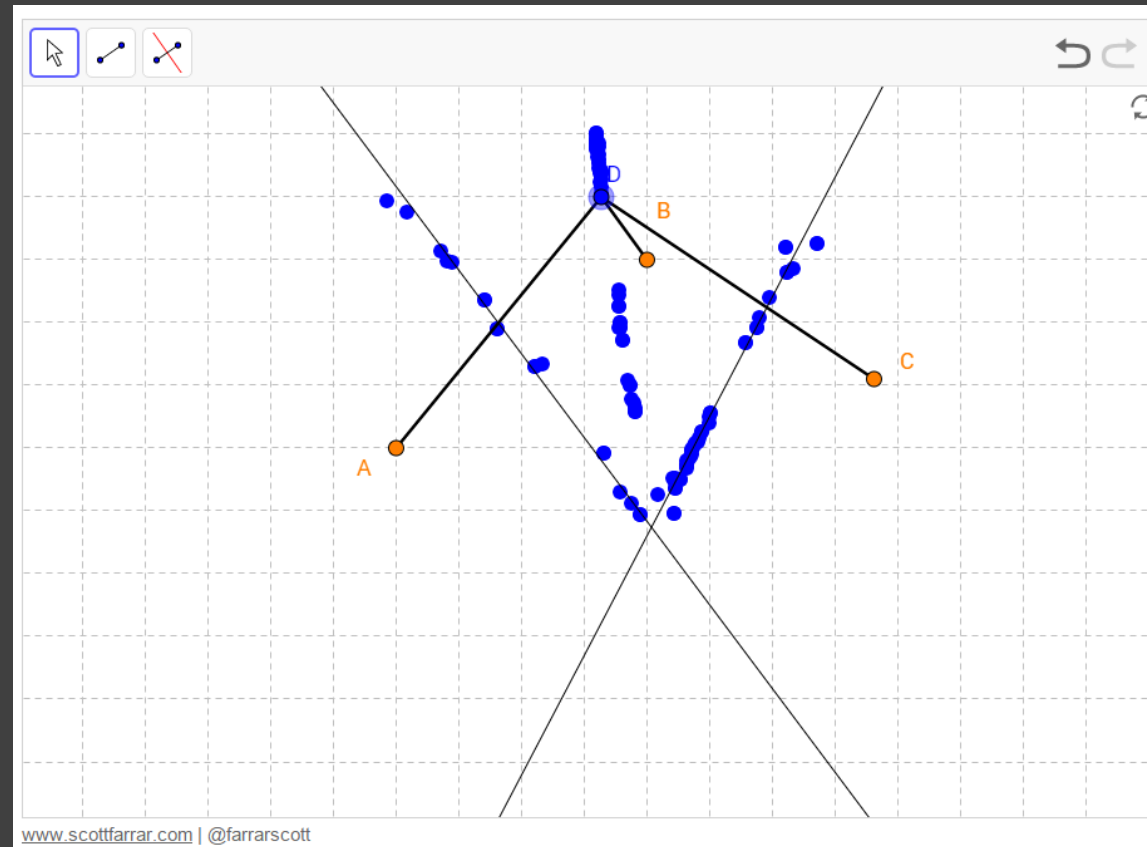
A manipulative is concrete

- sensory-interaction with a concept
- contains properties that support and build intuition.
- serves as an anchor point or foundation for analysis, deduction, and abstraction.

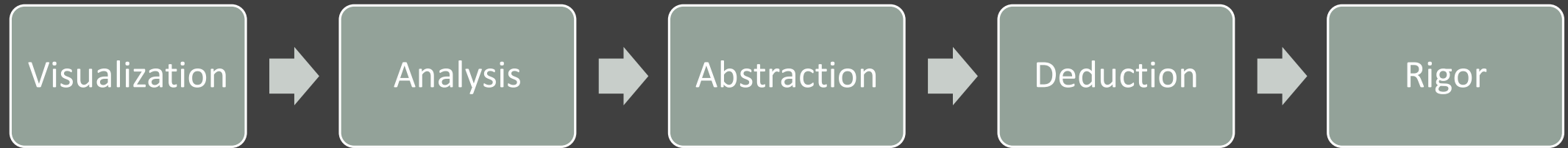
What's a “digital” manipulative?

- Concrete need not mean physical.
- Same concept of the circumcenter now using GeoGebra
- The computer offers us new opportunities

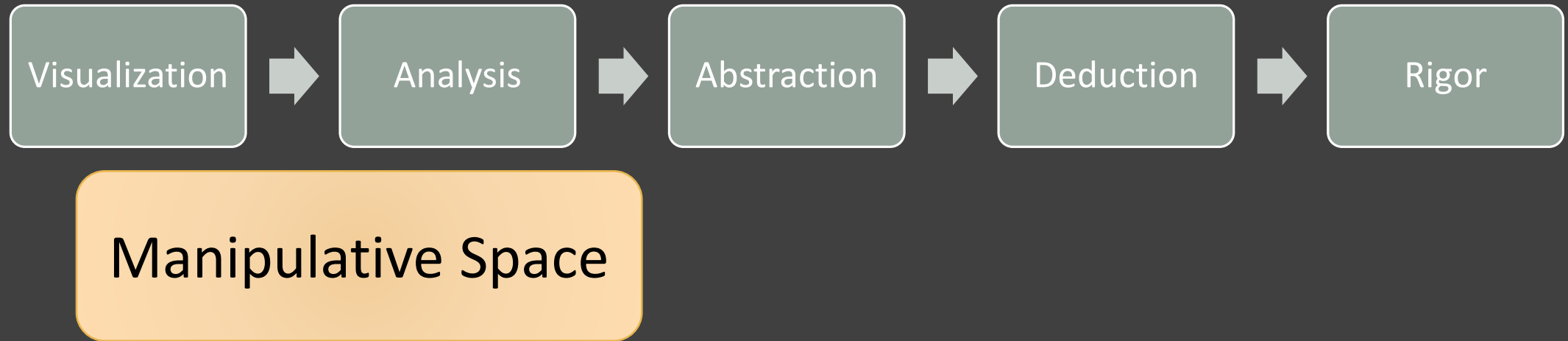




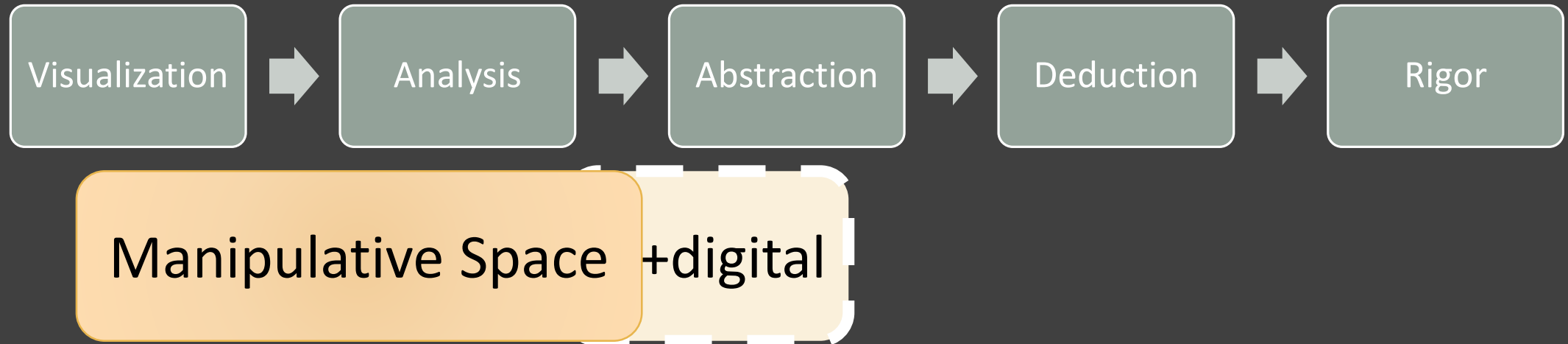
Van Hiele Model



Van Hiele Model



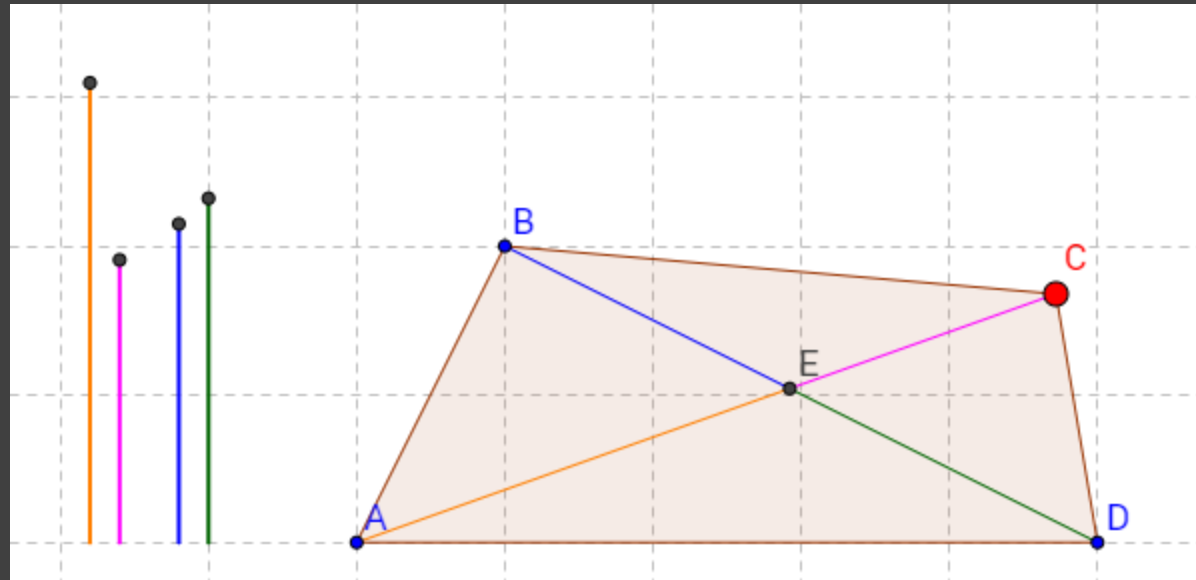
Van Hiele Model



Digital Manipulatives allow access to and movement on the “Ladder of Abstraction”

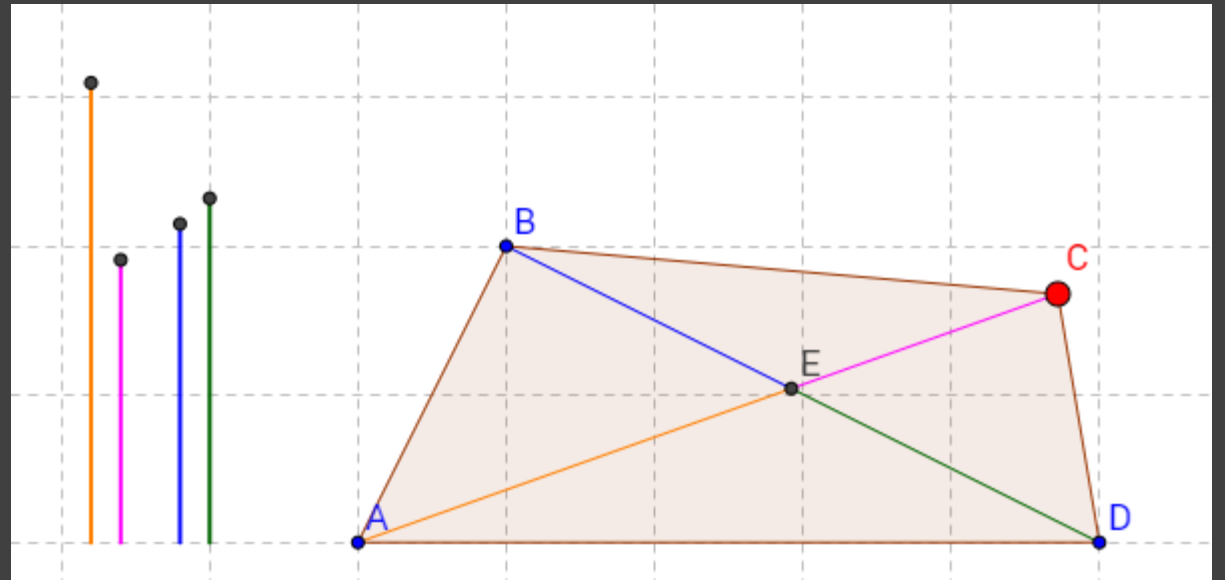
- Using Geogebra, we interacted with a **set** of triangles versus one on the patty paper.
- We had opportunities for conjecture and reasoning.
- We had agency, but structured agency.

Structured Opportunities for Reasoning



Structured Opportunities for Reasoning

By playing in this “sandbox”, the student can iterate through a controlled set of examples with agency. They can formulate and test conjectures – laying a bridgehead for deduction and rigor.



Digital Manipulatives allow access to and movement on the “Ladder of Abstraction”

“The computer manipulative can have just the mathematical features that developers wish it to have and just the actions on it that they wish to promote”

Sarama, J., & Clements, D. H. (2009). “Concrete” Computer Manipulatives in Mathematics Education. *Child Development Perspectives*, 3(3), 145–150.

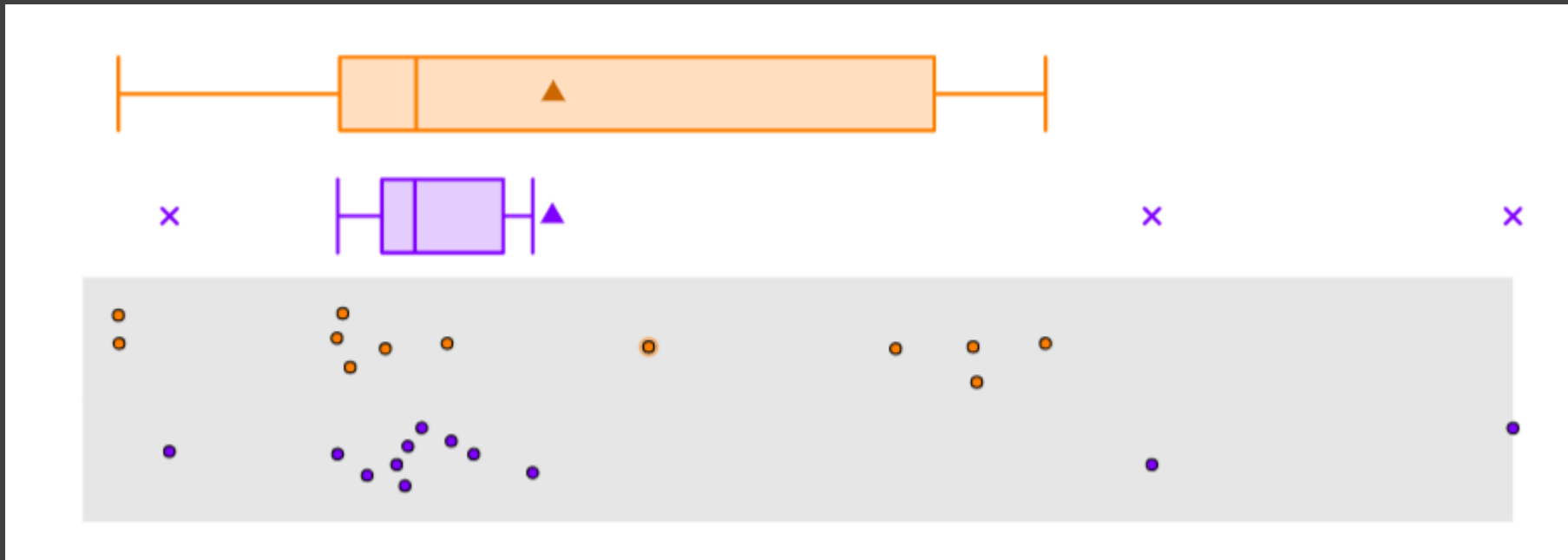
Bret Victor

*We stepped up a level of abstraction to **see a high-level pattern**, and then stepped down to **discover the explanation** for that pattern. I believe that this dance is where the deepest insights are born — not at any one level of abstraction, but in the **transitions between them**.*

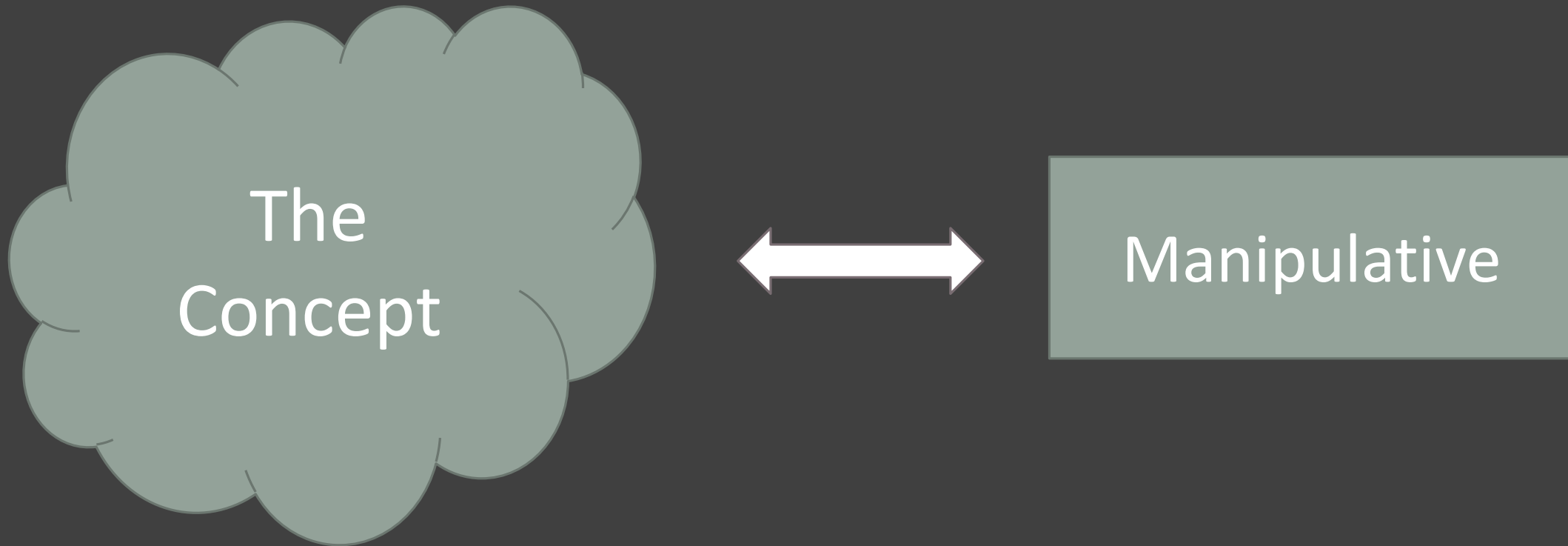
Victor, B. (2011). Up and down the ladder of abstraction. Retrieved from <http://worrydream.com/LadderOfAbstraction/>

Lets try one with Statistics

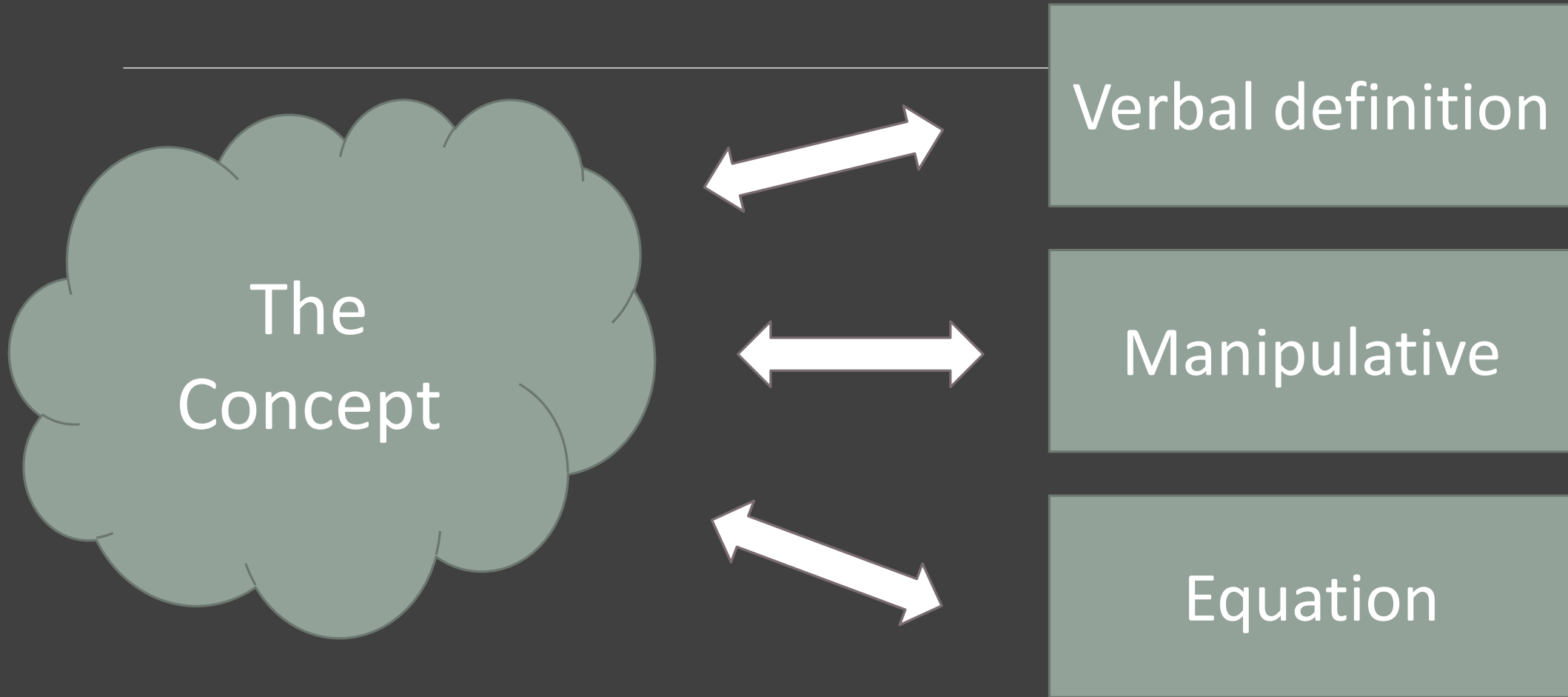
Can interact with the data via dragging and observation vs. computation



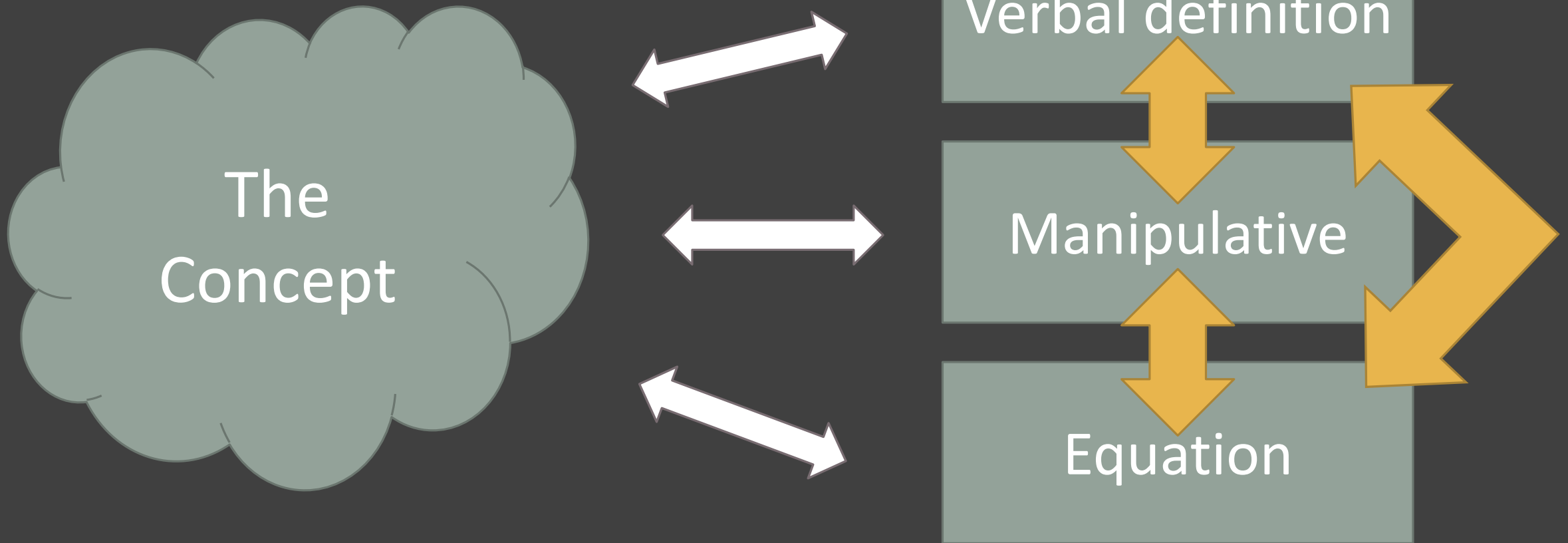
A manipulative provides a window into the concept



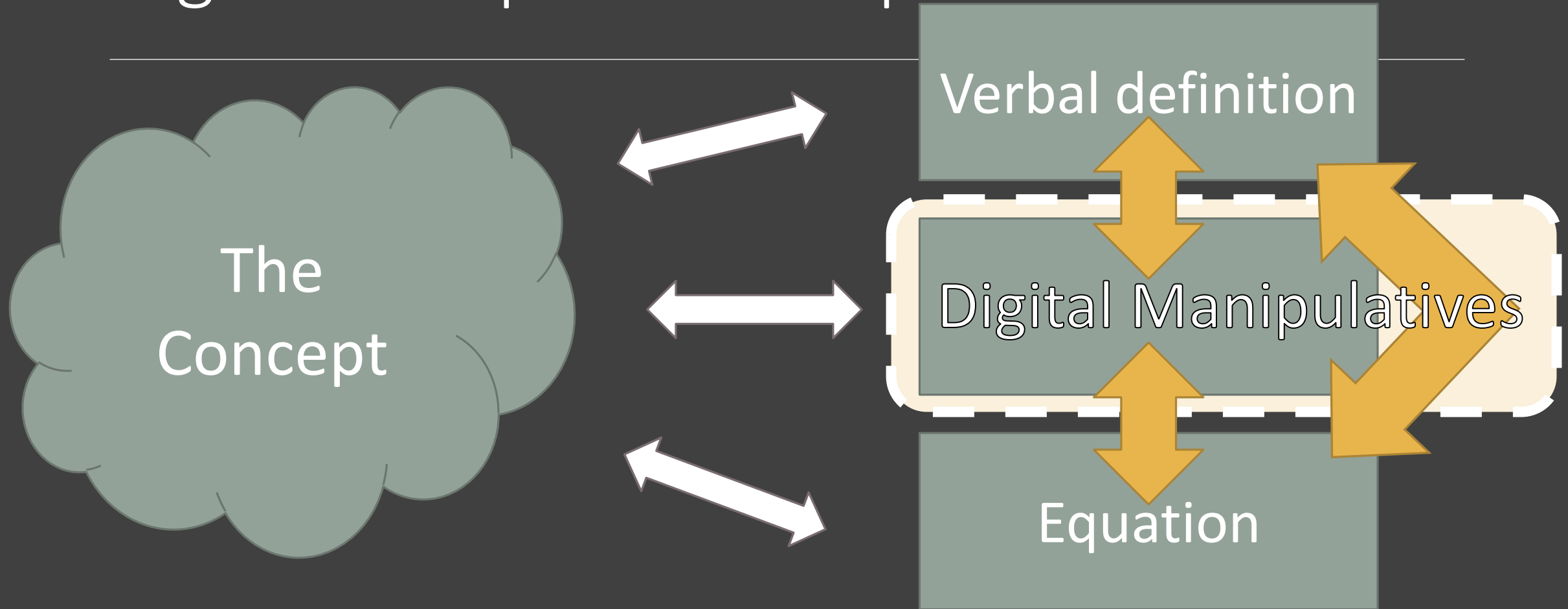
Additional representations allow additional windows



Connected knowledge



Digital Manipulatives expand the reach



Clements and McMillen

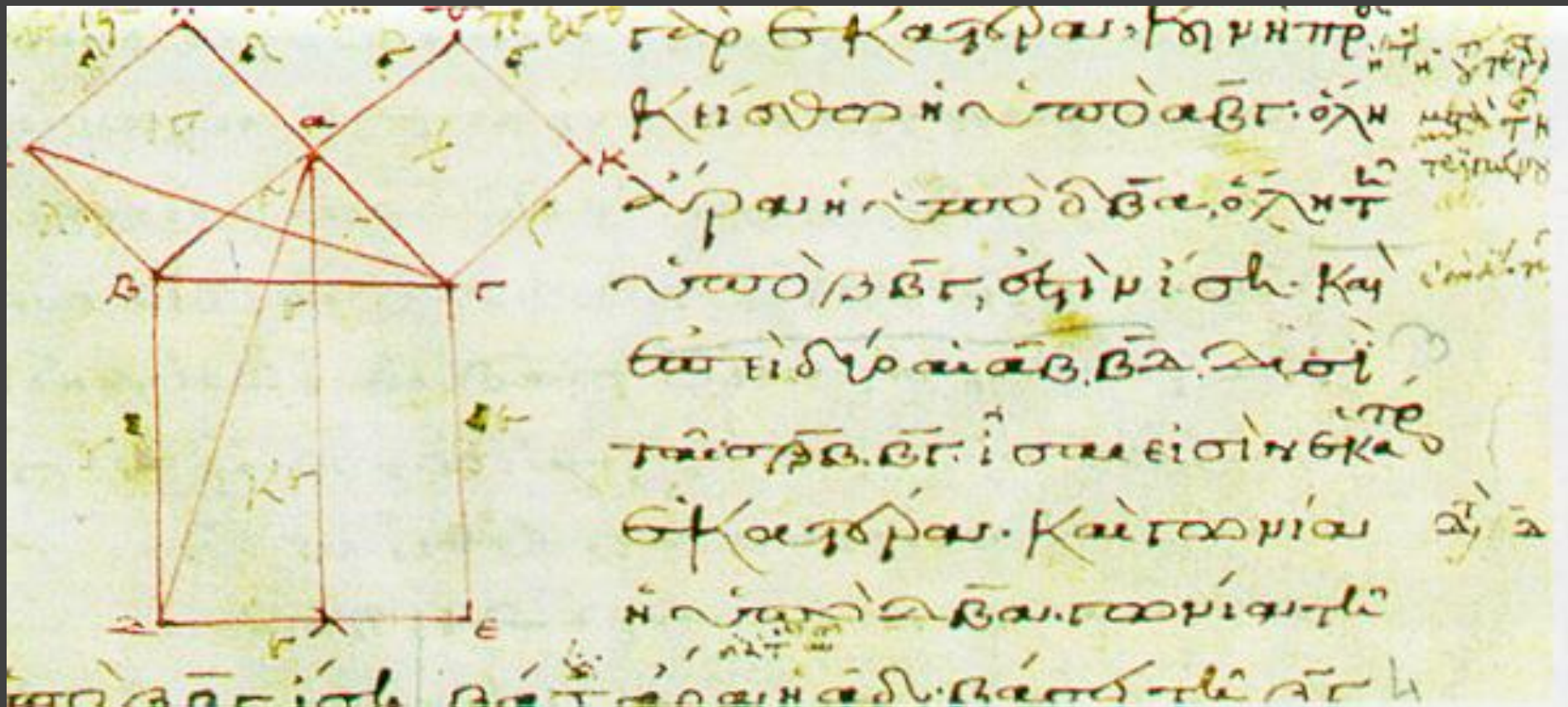
“Integrated-concrete thinking derives its strength from the combination of many separate ideas in an interconnected structure of knowledge.”

Clements, D. H., & McMillen, S. (1996). Rethinking “concrete” manipulatives. *Teaching Children Mathematics*, 2(5), 270–279.

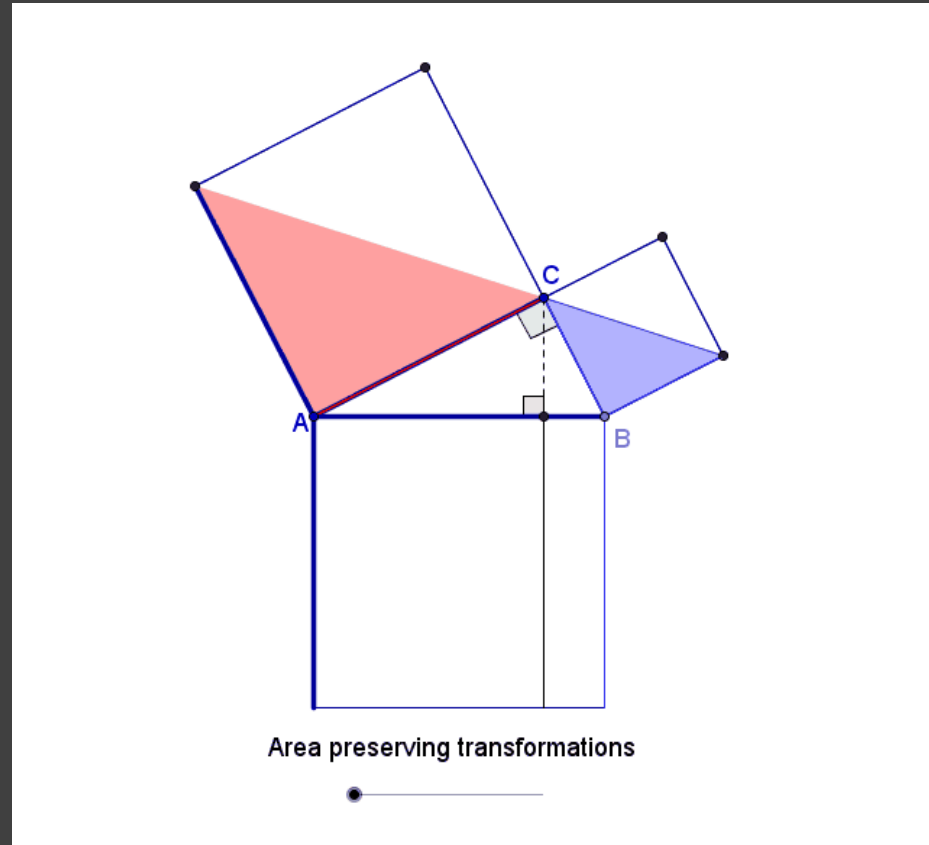
www.geogebra.org

Lets load it up

Euclid's Pythagorean Proof

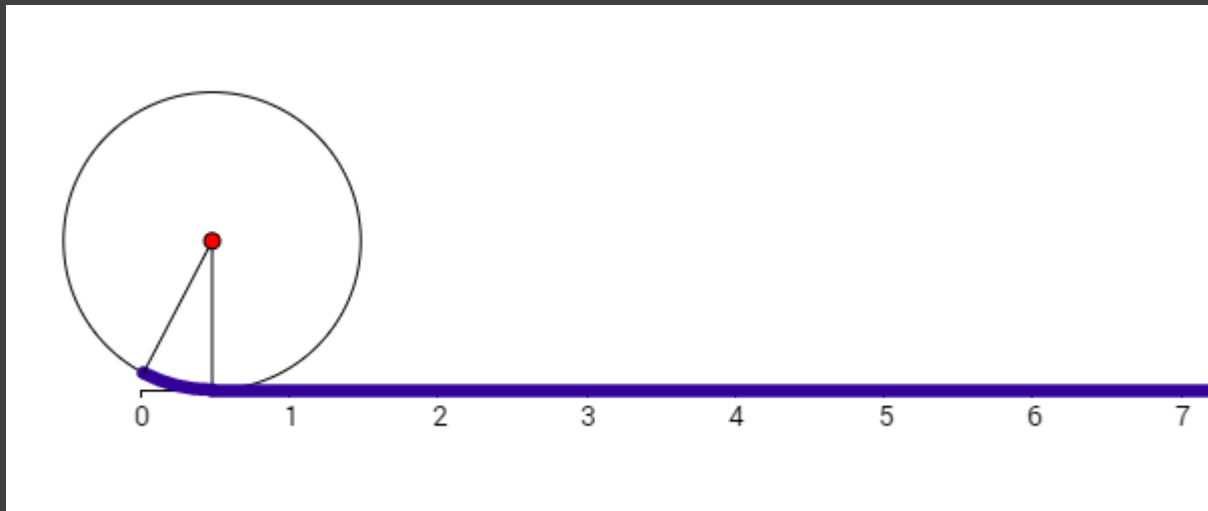


Euclid's Proof via GeoGebra



Radians and π

What is a radian?



Conclusion

*“Students learn about how the truth of a mathematical assertion gets established in mathematical discourse as they **zig-zag between their own observations and generalizations—their own proofs and refutations—revealing and testing their own definitions and assumptions as they go along.**”*

Lampert, M. (1990). When the Problem Is Not the Question and the Solution Is Not the Answer: Mathematical Knowing and Teaching. *American Educational Research Journal*, 27(1), 29–63.

Digital Manipulatives enable and support students to make these observations and generalizations in a highly customizable and portable environment.

Thanks!

- Scott Farrar

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www.scottfarrar.com

www.scottfarrar.com/quizlet2015